A new trilemma.

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One perennial unanswered question for debate among mathematics educators is that of 'what is mathematics?'. Polarised views range from those who point to traditional classical Euro-centric knowledge as the definitive answer to those who wish to widen the definition to include mathematics from every instance of mathematical activity. This paper explores the dilemma thus raised and suggests that the inclusion of a third element may help to clarify the issue.

Introduction

This paper had its origin in an essay for the Culture and Sociology of Mathematics Education paper at the University of Auckland. In the essay I looked back at the impact of a resource book *Kowhaiwhai - Geometry of Aotearoa* (McKenzie, 1989). At the time of its publication I believed that this would be the first of many bicultural mathematics resources in New Zealand, that the book was in line with the ideals of ethnomathematics, and that it would be a step towards changing the ways in which students and teachers, both Maori and non-Maori, would view mathematics. The essay dealt with the problems raised by the use of the resource; problems which highlighted one of the major conflicts within mathematics education - the conflict between traditional school mathematics and the mathematics found outside the classroom.

In this paper I intend to explore this conflict, and to suggest that the dilemma faced by educators wishing to make use of both types of mathematics is an oversimplification of the situation, and that there are at least three distinct types of mathematics available. I have called the issue raised by this third face of mathematics a *trilemma.* This proposal then raises further issues for educators and researchers:

- do all forms of mathematical activities qualify as material for mathematics research?
- can resources based upon differing types of mathematics be equally effective in the teaching of mathematics?
- are mathematical activities with a high cultural content open to all?

Mathematics and mathematics

When *Kowhaiwhai - Geometry of Aotearoa* was trialed at Huntly College in the Waikato region of New Zealand in 1986 I found that a number of students insisted that 'this isn't mathematics'. The course used kowhaiwhai, the rafter patterns which are a feature of the Maori meeting houses in New Zealand, to teach transformation geometry. Sometimes the objection was 'my father says this isn't mathematics!' At the time, I was scornful of such attitudes, on the basis that I was more likely to be in a position to judge what was or wasn't mathematics, but today I am not so certain.

A tension has existed among mathematics educators for a long time between those who incline to the position that mathematics is what is taught in schools and universities, and those who contend that mathematics is much wider than this, and must include all those areas where people use mathematical ideas in their everyday lives. Attempts to enlarge the definition of mathematics have met with considerable resistance and are a cause for debate among mathematical researchers

D'Ambrosio is usually credited with coining the term *ethnomathematics* (1984) and was amongst the first to define the expression. In the early evolution of the term he gave a definition “the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age, professional classes, and so on.” (D’Ambrosio, 1985). Such groups could be ethnic, but could also be social, such as the domain of darts-players, whose numerical computations and language of number are particular to those who take part in this sport. Such a broad definition will include many practices not normally regarded as mathematics. This does not mean that the practices do not contain elements easily identified as being mathematical, but that the underlying concepts do not fit into the structure commonly identified as mathematics.

Since D’Ambrosio’s early definition, mathematicians have continued to explore the term and what such a word might entail. Barton (1996a) describes ethnomathematics as a field of study which examines mathematical ideas in their cultural context. This involves challenging existing abstractions or concepts of mathematics. He notes that an alternative structure is not of itself ethnomathematical, but that the description refers only to the process of identification. He summarises this in a definition.

> Ethnomathematics is the field of study which examines the way people from other cultures understand, articulate and use concepts and practices which are from their culture and which the researcher describes as mathematical. (Barton, 1996a)

On this basis the *study of kowhaiwhai* is ethnomathematics, because I examined the rafter patterns and attempted to understand what the artist was doing, and was able to describe mathematical ideas. Studies of frieze patterns have also been done by Zaslavsky (1992) and Ascher (1991), the latter including kowhaiwhai as an example.

The Need for a Change

The interest in ethnomathematics is, at least in part, a reaction to what Joseph refers to as an ethnocentric bias (Joseph, 1995), a result of the hegemonic belief in the Eurocentric view of the development of mathematics. This view, he concludes, has lead to the concept of the subject as being deductive, abstract, a-prioristic, with axiomatic foundations. Joseph sees a multicultural approach to mathematics as part of a general strategy to make mathematics more accessible (Joseph, 1990), by countering the traditional view with one which recognises the contributions made by non-European mathematicians in the development of the subject.

Ethnomathematical study can arise for a number of reasons. Gerdes (1994) gives different types of mathematics which might be studied in this way, including:

- indigenous mathematics
- sociomathematics
- mathematics in the socio-cultural environment
- spontaneous mathematics
- oral mathematics
- oppressed mathematics
- non-standard mathematics
- frozen mathematics
- folk mathematics
- mathematics codified in know-hows.

All of these have been studied by a number of different researchers. Of particular interest to Gerdes was the way that mathematics changed in a country (Mozambique) as the political climate changed. Gerdes followed the evolution from an informal system aimed at inculcating obedience to the established tribal authority, through a Eurocentric colonial system which looked towards Portugal for examples to follow, into a revolutionary system where questions used examples of nurses at health posts, guerrilla fighters and literacy classes to meet the educational aims of the new government (Gerdes, 1981). In such case Gerdes noted that the mathematics was not culture-free, but that the culture of the period strongly influenced the content and pedagogy of the education system. Gerdes makes the distinction between the artisan who imitates a known production technique and the artisan who initially discovered the technique. The former, he claims, is not doing mathematics. However the latter (Gerdes, 1988) did do mathematics and was thinking in a mathematical way. Hence he sees the need for the mathematics classroom to be one in which the students re-invent the 'frozen' mathematics, and thus be stimulated.

These statements indicate that Gerdes sees the mathematical activities themselves as being examples of ethnomathematics, whereas Barton, D'Ambrosio and Ascher take the line that the activities provide for the *possibility* of ethnomathematics. In this line of thought, it is the study of the mathematical properties within the activity that is ethnomathematics, not the activity itself. This change in the meaning of the word *ethnomathematics* has been a gradual one, as the field of study has evolved.

The Three Face of Mathematics

It is commonplace nowadays to use the expressions *Mathematics* and *mathematics* to distinguish between the formal mathematics which is taught at school and onwards, and the informal mathematics which is used by people in everyday life and work. A third category (*premathematics)* has been added to these, as defined by Bishop (1988). Bishop lists six fields of activity which every culture develops:

- counting
- locating
- measuring
- designing
- playing
- explaining.

It is easy to see from the previous section how researchers have been so intent on valuing the activities of each culture that it has been an automatic process to seek mathematical content and then describe what is found as mathematical knowledge. Into this category I would place studies by Zaslavsky (1992) on counting and gaming, the extensive study by Lean (1992) on numbering systems in Papua New Guinea, and the recent work by Cooke (1995) on Yolgnu genealogy. These works all describe activities which can be placed into one of Bishop's six fields, and on this account I distinguish these from ethnomathematics.

There is a real danger with this approach of reading into an activity a lot more Mathematics than is actually present. Since each culture engages in premathematical activity, then every researcher who seeks evidence of premathematical activity will find it. If the researcher always finds mathematics, then the researcher who studies this mathematics is always a mathematician. It is not surprising therefore that mathematicians who lean towards the traditional view of the subject are unwilling to see its definition expanded, since the resulting scenario would resemble that sung by Don Alhambra in the Gilbert and Sullivan opera *The Gondoliers:* "If everybody is somebody, then no-one's anybody." But if this premathematics is mathematics in the early stages of development, then the study of premathematics differs from the study of ethnomathematics as described earlier. This is not to say that one form of mathematics is more important, or more valuable then another. This classification recognises the difference in what is being done in each type. Premathematics is the term for the activities done by a society or culture as essential building blocks in the development of that society or culture, mathematics with the small 'm' is the term for the mathematics which individuals and groups developed because of the needs of that individual or group, and a study of mathematical activities is what ethnomathematics does.

This mathematics differs from premathematics in that the areas of development will be largely determined by the environment within which that group is placed. Gerdes' study (1981) on liberation arithmetic is ethnomathematics, because the arithmetic has been used in a context for a new purpose (a political one). Thus a society which lives in a desert will develop spatial skills for survival to a far greater extent than others; a society which has permanent dwellings will develop building design skills to a far greater extent than nomads. A study into kowhaiwhai patterns (McKenzie, 1989) can be given as an example of ethnomathematics, and it differs from the premathematics examples given earlier because of a difference in what is being studied. Premathematical activities arise in every society because of basic needs, and these needs are common to all cultures. Ethnomathematical activities arise because of the interests of a particular culture. The mathematics of building a shelter is premathematical, because a shelter is an essential; the mathematics of systems of decoration is ethnomathematical because there comes a stage at which the artist is using and developing ideas for their own sakes. Ascher (1991) describes the rafter patterns as being formally structured, and as being the work of special craftsmen, part of a tradition passed on through generations. She notes that cultures such as the Maori have no intrinsic need for decorations with such a high degree of internal organisation. The patterns do not enhance the practical function of the adorned object, and therefore indicate an interest in spatial ordering. This move from essential to interest is what moves the activity from premathematics to mathematics, and the study from anthropology to ethnomathematics.

This distinction between premathematics and mathematics was described by Ascher when she moved from the premathematical limitations by saying

> "that if you go out and look for culturally embedded mathematical ideas, what you will find is counting. That is too small, it is much too small. Certainly mathematical ideas of number, logic, and spatial configuration organised into systems, include counting. But counting doesn't include all those wonderful things that are in mathematics and that are mathematical ideas" (Ascher and D'Ambrosio, 1994).

The third category, Mathematics, is that mathematics which we often identify as classroom mathematics, and differs in that mathematics is being studied as an entity, rather than a tool for some other purpose. Mathematics is sometimes as being described as what mathematicians do, and this definition represents a move away from mathematics as part of some larger activity, as in kowhaiwhai artistry, to Mathematics as being something to be studied for itself. D'Ambrosio claims (Ascher and D'Ambrosio, 1994) that you cannot separate a mathematics from the philosophy of the people who create and use it. He illustrates this by consideration of Greek attempts to trisect an angle by ruler and compass. He continues by emphasising that this attitude came because the scholars were interested in something other than solving a problem; it was not that they could not solve it, it was that they could not solve it under the conditions that they themselves had set. This is Mathematics - going beyond immediate needs to another level of reflection.

There is a continuum across the three categories, but some distinctions are clear. Thus a study of premathematics differs from a study of mathematics, which differs from a study of Mathematics.

Summary

If we accept the proposition that mathematics can be divided into three (at least) distinct categories: Mathematics, mathematics and premathematics, then we can give some distinguishing characteristics for each type.

1) Premathematics (as defined by Bishop)
   - restricted to the six areas of activity
   - static development
   - mathematics is used for the building of a society
   - the study is anthropology and is done by anthropologists
2) mathematics (as given in D'Ambrosio's early definition of ethnomathematics)
   - involving the development of mathematics in some area
   - dynamic development
   - users can experiment with ideas
   - mathematics is used to improve the quality of life (e.g. to make surroundings more attractive or to make someone a better trader
   - the study is ethnomathematics and is done by ethnomathematicians
3) Mathematics (as it is traditionally regarded)
   - new ideas are added to an existing body of knowledge
   - dynamic development
   - mathematics is often developed and studied for its own sake
   - the study is called mathematics and is done by mathematicians

While mathematicians who currently research premathematical activities may object to being described as anthropologists, my contention is that the term is a more accurate description of the work being done. I also recognise that many will argue the classifications which I have made in this paper, and there are certainly going to be many studies which will not fall neatly into one classification. These guidelines, however, can serve to make us aware of the distinct differences in the activities being studied. The three types do not form a hierarchy. If there is an ascending order in the sophistication of the mathematics used, that does not mean a corresponding increase in importance. In the past, ethnomathematics has too often been regarded as second class mathematics, (and presumably premathematics would be third class) but the difference is the role that the mathematics plays, not in their importance.

Conclusion

The trilemma for mathematics educators and researchers is to reconcile the differences between the three types of 'mathematics' so as to deliver the best to our students. If the three faces: Mathematics, mathematics, and premathematics are different, then the way in which we can use them will also be different. There is the potential for major problems in the misuse of mathematical and premathematical activities in misguided attempts to expand teaching into new areas within the classroom. When Cooke presented his paper at the Bridging Mathematics Conference at Batchelor in 1995, he asked a group of Yolgnu people who were present to comment, and received no reply; the group was at a loss as to know how to respond (Barton, 1996b). Barton argues that the study of gurrutu (Aboriginal kinship patterns) is not the same as the study of mathematical relationships. He adds:

> It would not be mathematics, but it was not intended to be. Similarly, what Michael Cooke explained to the conference was not gurrutu; it was some aspects of gurrutu which were recognisable as mathematical and which participants could relate to
> (Barton, 1996b).

This consequence of an attempt to merge cultural and mathematical activities can provide us with a clue to the trilemma of mathematics. Just as gurrutu is more than mathematical patterns, so kowhaiwhai are more than geometric designs. It is no easy thing to teach the entirety, to include all aspects of the activity, but it is more serious to focus solely on one facet. My students were nearly right. What I was teaching them was not Mathematics, what I was teaching was mathematics. What they expected, and what their parents expected, was Mathematics. This is not to say that we should, or must, be restricted in our teaching to the traditional Mathematics of old. There does need to be an acceptance of the wider definition to include other forms of 'mathematics', but we need to be aware that all 'mathematics' are not equal in the mathematical sense. We will find mathematical elements everywhere, but should not confuse the activity with the concept of Mathematics as something to be studied for its own sake. Premathematics is important, but not so much for the mathematics which we can find, as for the role it plays in the development of societies. My experience in teaching mathematics (small m) indicated that students were not inclined to accept mathematics but wanted Mathematics.

What is unknown is whether using mathematics is more effective than Mathematics in facilitating student learning. What is unknown is whether premathematics can be used successfully in mathematics education. If the categories are significantly different, it seems quite logical that teaching methodologies would also have to be altered for each type. There is considerable interest among teachers in the use of premathematics and mathematics. The task for researchers will be to determine whether these efforts can be productive and then how to maximise the effectiveness of new approaches to mathematics education.

My own experiences have indicated that the use of mathematical material can have both positive and negative effects. What I see as the next stage is a systematic investigation into the results of using material from the different types to teach and to measure the effectiveness of each. This will be a difficult task and a number of problems surface immediately. It will be necessary to find material from different resources to cover the same topics and to use these under controlled conditions. Parallel classes should provide some good evidence. However, the evidence and the objectives need to be carefully matched. Mathsphobia is part of our reality, and if an approach using mathematics or premathematics gives no better or even slightly worse understanding than a Mathematics approach, it will be important to also measure attitudes to see if some compensatory gains are made.

Care will be needed in choosing material which will not create problems of cultural ownership. Topics such as *gurrutu* and *kowhaiwhai* carry a great deal of cultural significance, and consultation with the cultural 'owners' will be essential before such material can be freely used. Consideration will therefore need to be given to the time required to use mathematical or premathematical activities to avoid 'westernising' the activity by omitting some facets of it to meet classroom time restrictions. Such a research programme may well require the resources of a number of interested parties, but the possibilities of widening the resources used in mathematics education provide good reason for taking this topic further.

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